

# Shot Noise Signatures of Charge Fractionalization in the $\nu = 2$ Quantum Hall edge

Mirco Milietari and Bernd Rosenow  
*Institut für Theoretische Physik, Universität Leipzig,  
 Vor dem Hospitaltore 1, 04103, Leipzig (Germany)*  
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We investigate the effect of non-equilibrium and interactions on shot noise in  $\nu = 2$  quantum Hall edges, where interactions between co-propagating edge modes are expected to give rise to charge fractionalization. Using the method of non-equilibrium bosonization, we find that even asymptotically the edge distribution function does not thermalize, but instead depends in a sensitive way on the interaction strength between edge modes. We compute shot noise and Fano factor from the asymptotic distribution function, and from comparison with a reference model of fractionalized excitations we find that the Fano factor can be close to the value of the fractionalized charge.

In contrast to three spatial dimensions, where excitations of an interacting many particle systems often carry the same quantum numbers as in the non-interacting case, interactions in 1d systems completely change the character of the excitation spectrum [1]. A prototype model for this physics is the Luttinger model, where electrons are no longer well defined quasi-particles, and where electronic excitations decompose into spin and charge parts moving with different velocities [2]

An important example of interacting 1d systems are the edge states of incompressible quantum Hall liquids [3, 4], where as a result of strong interactions charge fractionalization can occur [5–10] and manifest itself in shot noise [11–14]. For the case of filling fraction  $\nu = 2$ , there are two co-propagating chiral edge channels. In the presence of a short range interaction between them, a pulse of charge  $e$  injected into edge mode one at a first quantum point contact (QPC1) decomposes into a charge pulse and a neutral pulse. In the charge pulse, a charge  $e^*$  travels on mode two and  $e/2 + \sqrt{e^2 - 4(e^*)^2}$  on mode one [15]. In the neutral pulse, there is a charge  $-e^*$  on mode two and a charge  $e/2 - \sqrt{e^2 - 4(e^*)^2}$  on mode one. In this way, by exciting edge channel one via a partially transmitting QPC1, high frequency charge noise is generated on edge mode two [5]. A second QPC2, allowing for partial transmission of channel two, transforms the high frequency charge noise into zero frequency shot noise [6]. If a fractional charge  $e^*$  was scattered in the same way as an electron, one would expect a Fano factor  $e^*$  for the charge noise generated at QPC2.

There is an alternative way of looking at this problem, related to the concept of energy relaxation [16–19]. Interactions play a crucial role in the thermalization process that drives a system through states described by the Gibbs equilibrium ensemble. Generically, the dynamics is only constrained by two integrals of motion, total energy and total particle number. Integrable models like the  $\nu = 2$  quantum Hall edge have infinitely many integrals of motion, and therefore it is not clear if an equilibrium state can ever be reached [20]. If the two edge modes are driven out of equilibrium with respect to one another, the system relaxes towards a non-thermal steady

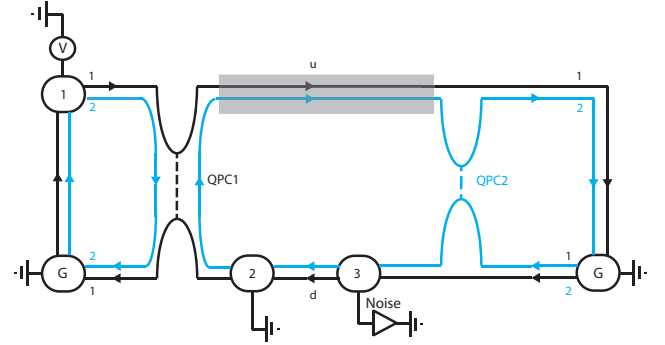


FIG. 1: (color online) Sketch of a  $\nu = 2$  Hall bar with a QPC1, where inner modes ("2", light blue) are fully reflected, while partial transmission of outer edge modes ("1", black) is possible. At QPC2, the opposite situation is realized. The shaded area is the interaction region, where partial energy relaxation takes place. The upper edge is biased with voltage  $V$  at contact 1, current noise is measured at contact 3.

state, whose distribution function determines shot noise at QPC2. The corresponding Fano factor depends on the strength of the interaction between the two edge modes, and in general neither agrees with the fractional charge  $e^*$  introduced above, nor with the result for two equilibrated edge modes. For the special case of a half open QPC1 however, the Fano factor is close  $e^*/e$ , suggesting an interpretation in terms of charge fractionalization.

We consider the setup Fig. 1 where a Hall bar is pinched by two QPCs. The outer edge mode is labeled "1" and the inner one "2". The top and bottom edges originate at zero temperature from reservoirs at voltages  $V_1 = V$  and  $V_2 = 0$ . At QPC1, the outer modes are partially transmitted with probability  $a$ , while the inner ones are fully transmitted; as a consequence, only the outer mode becomes noisy. After QPC1, the two edge modes interact over some distance (shaded area in Fig. 1 with length  $\ell$ ) before reaching QPC2. Here, the outer modes are fully reflected while the inner ones are partially transmitted with probability  $p$ . Current noise is then measured at contact 3. Using the recently de-

veloped non-equilibrium bosonization technique [21, 22] within a quantum-quench model [23–25], we compute the shot noise at QPC2, with particular emphasis on its dependence on the strength of the interaction between the two edge modes.

The edges and QPC2 are described by the following Hamiltonian ( if not otherwise stated, we adopt units with  $\hbar = k_B = 1$  ) :

$$\begin{aligned}\mathcal{H}_\eta &= 2\pi \int_x (v_1 \rho_{1,\eta}^2(x) + v_2 \rho_{2,\eta}^2(x) + v_{12} \rho_{1,\eta}(x) \rho_{2,\eta}(x)) \\ \mathcal{H}_{QPC2} &= t_2 \psi_{2,u}^\dagger(x) \psi_{2,d}(x) + h.c.\end{aligned}\quad (1)$$

Here,  $\mathcal{H}_\eta$  describes chiral modes with velocities  $v_1, v_2$ , and with a local interaction  $v_{12}$  between them.  $\mathcal{H}_{QPC2}$  describes scattering of electrons at QPC2 with  $t_2$  the scattering amplitude. The fields  $\rho_{i,\eta}(x)$  in (1) describe density fluctuations, they are related to bosonic displacement fields by  $\rho_i(x) = \partial_x \phi_i(x)/2\pi$ ; here "i" labels different edge modes, and  $\eta = u, d$  the upper and lower edge. The bosonic fields have commutators  $[\phi(x), \phi(y)] = i\pi \text{sign}(x - y)$ , and the fermionic field is represented as  $\psi_\eta(x) = (2\pi\alpha)^{-1/2} e^{i\phi_\eta(x)}$  with  $\alpha$  denoting a short distance cutoff on the scale of the magnetic length. For later reference, we decompose the bosonic fields as  $\phi_\eta(x) = \varphi_\eta(x) + \varphi_\eta^\dagger(x)$ ,  $\varphi_\eta(x) = \sum_{q>0} \sqrt{2\pi/qL} e^{-iqx/2} e^{is_\eta q x} b_\eta(s_\eta q)$ , where  $s_\eta = \pm 1$  respectively for right (u) and left (d) movers, and  $b^\dagger, b$  are canonical bosonic operators.

Following [18], we do not model QPC1 explicitly but instead consider its effects on the downstream electron distribution of mode (1u), which is modeled as a "double step" function

$$f(\epsilon) = a\theta(-\epsilon + \mu_1) + (1-a)\theta(-\epsilon + \mu_2), \quad (2)$$

where  $\mu_1 = (1-a)eV$  and  $\mu_2 = -aeV$  (assuming  $eV > 0$ ) are chosen such that the average density in mode (1u) corresponds to zero bias. This choice for  $f$  is made to ensure that the interaction  $v_{12}$  does not give rise to a density shift in mode (2u).

Next, we consider the effects of the inter-mode interaction on the distribution functions by formally turning on the interaction right after QPC1. We use the model of a quantum quench, where the interaction is suddenly turned on at time  $t = 0$ , thus changing the coupling constant  $v_{12}$  from zero to a finite value. The full interacting problem can now be diagonalized by means of a Bogoliubov transformation  $M$ . For co-propagating states ( $v_1 v_2 > 0$ ),  $M$  can be represented by the following rotation matrix in operator space :

$$M = \begin{pmatrix} \cos \theta & \sin \theta \\ -\sin \theta & \cos \theta \end{pmatrix}, \quad (3)$$

allowing to express  $\mathcal{H}$  in terms of new (rotated) fields  $\beta_{i,q} = \sum_j M_{ij} b_{j,q}$ . The mixing angle  $\theta$  expresses the

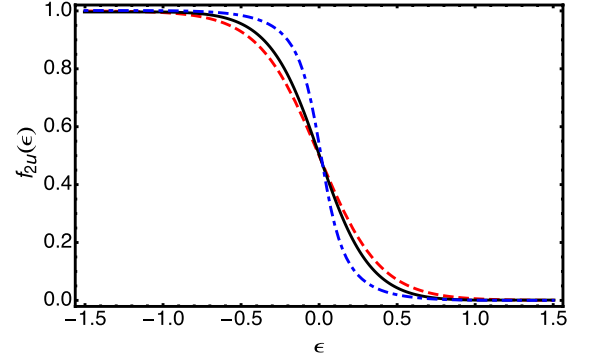


FIG. 2: (color online) Steady state distribution of edge mode (2u) asymptotically far away from the QPC1. (black full line) Nonequilibrium distribution obtained from Eqs. (7,9) by summing over all cumulants. (blue dash-dotted line) distribution obtained by retaining only the gaussian term. (red dashed line) fully equilibrated distribution at effective temperature  $T^* = eV \sqrt{(3/2)a(1-a)}/\pi$ . The mixing angle is  $\theta = 0.47$ , and the transmission probability of QPC1 is  $a = 1/2$ .

strength of the interaction through the relation  $\tan 2\theta = v_{12}/(v_1 - v_2)$ . At this point the rotated operators evolve in the Heisenberg picture as  $\beta_{iq}(t) = e^{-iq\tilde{v}_i t} \beta_{iq}(t=0)$ , where the new velocities are given by  $\tilde{v}_{1(2)} = v_{1(2)} \cos^2 \theta + v_{2(1)} \sin^2 \theta \pm \frac{1}{2} v_{12} \sin 2\theta$ .

As a final step, we undo the Bogoliubov transformation in order to express the  $\beta_{iq}(t_0)$  in terms of the original basis. As a result, we obtain a relation between the bosonic operators at  $t_0 > 0$  and those at  $t = 0$  :

$$\begin{aligned}b_{1q}(t_0) &= u_q(t_0)b_{1q} + s_q(t_0)b_{2q} \\ b_{2q}(t_0) &= s_q(t_0)b_{1q} + v_q(t_0)b_{2q}\end{aligned}\quad (4)$$

where  $b_{iq} \equiv b_{iq}(t=0)$ . Now all the time dependence is encoded in the coefficients

$$\begin{aligned}u_q(t_0) &= \cos^2 \theta e^{-iq\tilde{v}_1 t_0} + \sin^2 \theta e^{-iq\tilde{v}_2 t_0} \\ v_q(t_0) &= \cos^2 \theta e^{-iq\tilde{v}_2 t_0} + \sin^2 \theta e^{-iq\tilde{v}_1 t_0} \\ s_q(t_0) &= \frac{1}{2} \gamma_\theta (e^{-iq\tilde{v}_1 t_0} - e^{-iq\tilde{v}_2 t_0}),\end{aligned}\quad (5)$$

where  $\gamma_\theta = \sin 2\theta$ . To leading order in the tunneling amplitude  $t_2$ , the noise in the tunnel current at QPC2 can be expressed in terms of greater (lesser) Green functions  $G_{i,\eta}^{>(<)}(\epsilon)$  [26] as

$$S_{\omega \rightarrow 0} = \frac{2e^2}{h} \frac{|t_2|^2}{2\pi} \int_\epsilon G_{2u}^{<}(\epsilon) G_{2d}^{>}(\epsilon) + G_{2d}^{<}(\epsilon) G_{2u}^{>}(\epsilon), \quad (6)$$

with  $G^{<}(\epsilon) = G^{>}(-\epsilon)$ . Using the boson representation of electron operators, we can compute  $G_{2u}^{>(<)}(\tau)$  of the fully interacting edge mode. Due to the non-equilibrium distribution of edge mode (1u), calculating the expectation value of a product of bosonic exponents is highly non trivial. Here we discuss the results for the "long time

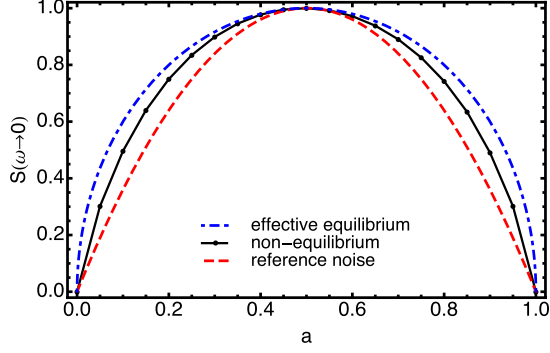


FIG. 3: (color online) Shot noise after QPC2 as a function of transparency  $a$  of QPC1, normalized to one at  $a = 1/2$ , for a mixing angle  $\theta = 0.47$ . Comparison of full non-equilibrium result (full black line), reference noise of noninteracting electrons (dashed red line), and noise in a fully equilibrated thermal state (dash-dotted blue line).

limit” of the Green function, in which the system reaches a non-equilibrium steady state:

$$\begin{aligned} G_{2u}^<(\tau) &= \langle \tilde{\psi}_{2u}^\dagger(t_0 + \tau, x) \tilde{\psi}_{2u}(t_0, x) \rangle \\ &= G_0^<(\tau) \langle e^{\sum_q \lambda_{1u}^*(q, t_0, \tau) b_{1u, q}^\dagger} e^{-\sum_q \lambda_{1u}(q, t_0, \tau) b_{1u, q}} \rangle, \\ G_0^<(\tau) &= \frac{1}{2\pi\alpha} \frac{1}{(-i\tilde{v}_1\tau + \alpha)^{\gamma_\theta^2/2}} \frac{1}{(-i\tilde{v}_2\tau + \alpha)^{1-\gamma_\theta^2/2}}. \end{aligned} \quad (7)$$

Here,  $G_0^<(\tau)$  is the equilibrium Green function of edge mode 2 in the presence of interactions. All the information about non-equilibrium effects is contained in the average over bosonic coherent states in Eq. (7), where  $\lambda(q, t_0, \tau) = i s_q(t_0)(2\pi/qL)^{1/2} e^{iq(x-\tilde{v}_1 t_0)} (e^{-i\tilde{v}_1 q\tau} - 1)$ .

As emphasized in [21], non-equilibrium effects make the theory non-Gaussian, and higher order cumulants appear in the evaluation of the above expectation value. In order to compute the expectation value over the non-equilibrium state, we reformionize the bosonic operators introducing new fermionic operators [27]:

$$\begin{aligned} b_{1u}^\dagger &= (2\pi/qL)^{1/2} \sum_k \tilde{c}_{1u, k+q}^\dagger \tilde{c}_{1u, k}, \\ b_{1u} &= (2\pi/qL)^{1/2} \sum_k \tilde{c}_{1u, k-q}^\dagger \tilde{c}_{1u, k}. \end{aligned} \quad (8)$$

Since the bosonic operators describe free particle-hole excitations, also the  $\tilde{c}$ -operators are free and therefore can be connected to the incoming states via a scattering matrix. Then, the desired expectation values of products of Fermi operators can be evaluated by using Wick's theorem and an appropriate fermionic density matrix  $\tilde{\rho}_{1u}$ . The crucial step now consists in noticing that the computation of higher order cumulants is similar to the problem of full counting statistics, and using Klich's trace formula [22, 28] they can be expressed in terms of a Fredholm determinant of the Toeplitz type, normalized to its zero

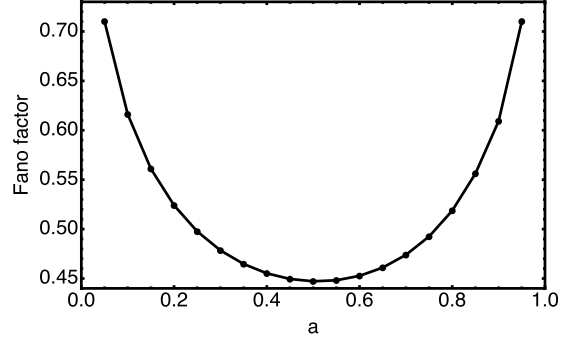


FIG. 4: Fano factor as a function of transparency of QPC1 for the mixing angle  $\theta = 0.47$ . At  $a = 1/2$  the Fano factor is  $F = 0.45$ .

temperature, equilibrium value

$$\bar{\Delta}_\tau(\delta) = \frac{\det[1 + (e^{-i\delta\tau} - 1)f(\epsilon)]}{\det[1 + (e^{-i\delta\tau} - 1)\theta(-\epsilon)]}, \quad (9)$$

where  $f(\epsilon)$  is given by Eq. (2). The scattering phase  $\delta_\tau = \pi\gamma_\theta\omega_\tau(t_0, 0)$  contains information about the inter-edge interaction, and the window function

$$\begin{aligned} \omega_\tau(t_0, 0) &= \theta(x - (\tilde{v}_1 + \tilde{v}_2)t_0) - \theta(x - (\tilde{v}_1 + \tilde{v}_2)t_0 - \tilde{v}_2\tau) \\ &\quad - \theta(x - 2\tilde{v}_2 t_0) + \theta(x - \tilde{v}_2(2t_0 + \tau)) \end{aligned} \quad (10)$$

restricts the action of the determinant over a time range  $\tau$ . The window function represents a combination of two square pulses which separate on a time scale  $\tau \ll \ell/(\tilde{v}_2 + \tilde{v}_1)$ . In this limit, the expectation value of bosonic coherent states factorizes into a product of two determinants having the same scattering phase, and we can rewrite Eq. (7) as  $G_{2u}^<(\tau) = G_0^<(\tau) \bar{\Delta}_\tau^2(\delta)$ . The determinant Eq. (9) can be evaluated numerically by carefully defining a regularization scheme [22].

Finally, the lesser Green function  $G_{2d}^<(\epsilon) = \theta(-\epsilon)/\tilde{v}_1^{\gamma_\theta^2/2}\tilde{v}_2^{1-\gamma_\theta^2/2}$  is easily evaluated due to its equilibrium nature. Fourier transforming Eq. (7) into energy space, we can compute the distribution function at QPC2; as a consequence of interactions, the distribution function is broadened from a single step (see Fig. 2). However, it does not have the same functional form as a Fermi distribution, but rather describes a true non-equilibrium steady state. The distribution obtained by only retaining the Gaussian term in the cumulant expansion clearly deviates from the full one, making evident the necessity for including higher order terms. The non-equilibrium distribution also deviates from an equilibrium Fermi distribution with effective temperature  $T^* = eV\sqrt{(3/2)a(1-a)}/\pi$ , obtained by assuming that the two edge modes fully equilibrate and that each of them carries half the energy flux injected into the upper edge via QPC1.

In Fig. (3), we display the dependence of low frequency noise computed from Eq. (6) on the transmission  $a$  of

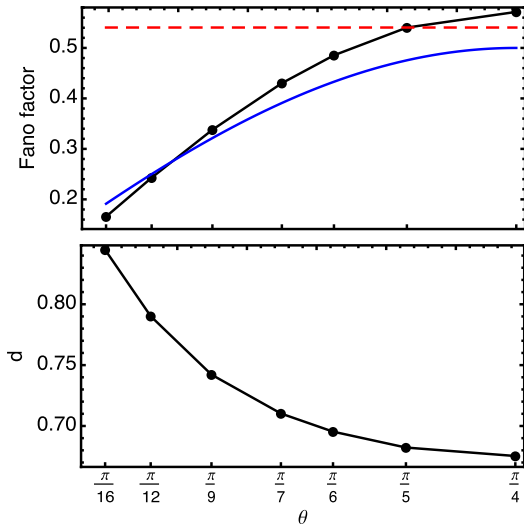


FIG. 5: (color online) Upper panel: Fano factor as a function of the mixing angle for transmission  $a = 1/2$  of QPC1. (red, dashed) Fully equilibrated edge,  $F$  is independent of interactions. (black, dotted) Full non-equilibrium situation. (blue, continuous) Reference model diluted system of fractional charges ( $F = (1/2) \sin 2\theta$ ). Lower panel: Fitting the non-equilibrium noise displayed in Fig. 3 by a function proportional to  $(a(1-a))^d$ , the parameter  $d$  depends on the mixing angle  $\theta$ .

QPC1, normalizing the noise by its value at  $a = 1/2$ . One clearly sees that it deviates from both the standard free fermion dependence  $a(1-a)$ , and from the equilibrium result with  $\sqrt{a(1-a)}$ .

To put the strength of the noise at QPC2 in perspective, we define a reference noise expected for non-interacting electrons tunneling through both QPC1 and QPC2 along a single edge, obtained by using the distribution Eq. (2) in Eq. (6)

$$S_{\text{ref}}(\omega \rightarrow 0) = 4epa(1-a)I \quad \text{with} \quad I = \frac{e^2}{h}V. \quad (11)$$

Here, the transmission probability of QPC2 is  $p = |t_2|^2 / 2\pi \tilde{v}_1^{\gamma_\theta^2} \tilde{v}_2^{2-\gamma_\theta^2}$ .

Defining a Fano factor  $F = S/S_{\text{ref}}$  as the ratio between the noise obtained from Eq. (6) and the reference noise Eq. (11) (see Fig. 4), we can make contact with the concept of fractional charges described in the introduction. Assuming that the non-equilibrium state of mode  $(2u)$  could be described by a diluted system of fractional charges  $e^* = (e/2) \sin 2\theta$  [15], it is plausible that the noise would be given by Eq. (11) when replacing the charge  $e$  by  $e^*$ , giving rise to a Fano factor  $e^*/e$  (strictly speaking, there are two charges  $e^*$  and  $-e^*$  associated with each electron, but on the other hand the “currentless” noise of Eq. (11) already contains contributions of both electrons and holes, so at least for the specific value of  $a = 1/2$  this would amount to double counting). In Fig. 5, the Fano

factor is shown as a function of mixing angle for the specific transmission  $a = 1/2$  of QPC1. For this value of  $a$ , there is a surprisingly good agreement between the value  $e^*/e = (1/2) \sin 2\theta$  [15] and  $F$  of the full non-equilibrium noise, suggesting that the Fano factor can be interpreted as being due to formation of fractionalized charges in the  $\nu = 2$  quantum Hall edge.

We find that the zero frequency noise power depends in a singular way on  $a$  in the limit  $a \ll 1$  [29]. To obtain the noise in this limit, the functional determinant can be approximated by its long time asymptotics (valid for  $eV\tau \gg 1$ )  $\tilde{\Delta}_\tau(\delta) \simeq \exp(-|\tau|/(2\tau_\phi))$ , where the dephasing rate  $\tau_\phi^{-1} = -(eV/2\pi) \log[1 - 4a(1-a) \sin^2(\delta/2)]$ . Knowledge of  $\tilde{\Delta}_\tau(\delta)$  for large times allows to accurately calculate the distribution function of mode  $(2u)$  for energies  $\epsilon \ll eV$ . However, for  $a \ll 1$  the distribution function only deviates from a step function on the scale  $aeV$ , such that the long time asymptotics allows an exact calculation of the distribution function. Using Eq. (6) and taking the  $a \ll 1$  limit, we find  $S(\omega \rightarrow 0) \simeq 8pa \log(1/a) \sin^2(\delta/2) eV (e^2/h\pi^2)$ .

A useful way to characterize the nonlinear dependence of experimentally measured shot noise on the transmission probability  $a$  of QPC1 is by fitting it to a function proportional to  $(a(1-a))^d$  [30]. For the reference noise of Eq. (11),  $d$  is trivially equal to unity. For “thermal” noise generated by the effective equilibrium temperature  $T^*$ , one finds  $d = 0.5$ . For the full equilibrium noise, we find that its dependence on  $a$  can be well fitted by the above power law, and that  $d$  varies from  $d = 0.85$  for  $\theta = \pi/16$  to  $d = 0.68$  for  $\theta = \pi/4$ , see Fig. 5. In this way, from knowledge of  $d$  the mixing angle  $\theta$  can be inferred, without making use of the Fano factor.

In summary, due to the joint effect of interactions and non-equilibrium, the distribution function of an originally unbiased, zero temperature mode  $(2u)$  interacting with a noisy mode  $(1u)$  evolves towards a non-thermal steady state that depends on the interaction strength in an essential way. Comparing the shot noise and Fano factor from our numerically exact calculation with a simple model of charge fractionalization, we find that the Fano factor can indeed be interpreted in terms of charge fractionalization in the  $\nu = 2$  quantum Hall edge.

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- [29] After completion of this work, we have become aware of the preprint by I.P. Levkivskyi and E.V. Sukhorukov, arXiv:1206.4566, where shot noise due to interaction effects in a  $\nu = 2$  quantum Hall edge is studied using the formalism of non-equilibrium full counting statistics. Levkivskyi and Sukhorukov emphasize the nonanalytic dependence  $S \propto a \ln 1/a$  of the noise on the transmission probability of QPC1 in the limit of small  $a$ , obtained by an asymptotically exact calculation.
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